# $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$, a new noncentrosymmetric sodium borate 

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#### Abstract

$\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}(\mathrm{~B} / \mathrm{Na}=2.17)$ single crystals were obtained by heating, melting and appropriately cooling borax, $\mathrm{Na}_{2}$ $\left[\mathrm{B}_{4} \mathrm{O}_{5}(\mathrm{OH})_{4}\right] \cdot 8 \mathrm{H}_{2} \mathrm{O}$. Its formula has been determined by the resolution of the structure from single-crystal X-ray diffraction data. The compound crystallizes in the noncentrosymmetric orthorhombic $I b a 2$ space group, with the following unit cell parameters: $a=33.359(11) \AA, b=9.554(3) \AA, c=10.644(4) \AA ; V=3392.4(19) \AA^{3} ; Z=8$. The crystal structure was solved from 3226 reflections until $R_{1}=0.0385$. It exhibits a three-dimensional framework built up from $\mathrm{BO}_{3}$ triangles ( $\Delta$ ) and $\mathrm{BO}_{4}$ tetrahedra (T). Two kinds of borate groups can be considered forming two different double $\mathrm{B}_{3} \mathrm{O}_{3}$ rings: two $\mathrm{B}_{4} \mathrm{O}_{9}$ (linkage by two boron atoms) and one $\mathrm{B}_{5} \mathrm{O}_{11}$ (linkage by one boron atom); the shorthand notation of the new fundamental building block ( FBB ) existing in this compound is: 13: $\infty^{3}[(5: 3 \Delta+2 \mathrm{~T})+2(4: 2 \Delta+2 \mathrm{~T})]$. The discovery of this new borate questions the real number of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ varieties. The existence of $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}(\mathrm{~B} / \mathrm{Na}=2.17)$ and of another recently discovered borate, $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}\left(\mathrm{~B} / \mathrm{Na}=2.33 ; \mathrm{FBB} 7: \infty^{3}[(3: 2 \Delta+\mathrm{T})+(3:\right.$ $\Delta+2 \mathrm{~T})+(1: \Delta)]$, with a composition close to the long-known borate $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}\left(\mathrm{~B} / \mathrm{Na}=2\right.$; FBB 8: $\infty^{3}[(5: 3 \Delta+2 \mathrm{~T})+(3: 2 \Delta+\mathrm{T})]$, may explain the very complex equilibria reported in the $\mathrm{Na}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ phase diagram, especially in this range of composition. (C) 2004 Elsevier Inc. All rights reserved.


Keywords: Borate; Sodium borate

## 1. Introduction

During the last 2 years, two new anhydrous sodium borates have been obtained, $\beta-\mathrm{Na}_{2} \mathrm{~B}_{8} \mathrm{O}_{13}$ [1] and $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}$ [2], although these compounds were extensively studied as main constituents of borosilicate glasses. In the successive versions of $\mathrm{Na}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ binary phase diagram, numerous compounds with several crystalline forms were reported [3-6]. However, only some of them were clearly identified because their structures were determined: $\mathrm{Na}_{3} \mathrm{BO}_{3}$ [7], $\mathrm{Na}_{4} \mathrm{~B}_{2} \mathrm{O}_{5}$ [8], $\mathrm{NaBO}_{2}$ [9], $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ [10], $\alpha-\mathrm{NaB}_{3} \mathrm{O}_{5}$ [11], $\beta-\mathrm{NaB}_{3} \mathrm{O}_{5}$ [12], and $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{8} \mathrm{O}_{13}$ [13-15]. $\mathrm{Na}_{2} \mathrm{O} \cdot 2 \mathrm{~B}_{2} \mathrm{O}_{3}\left(\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}\right)$ borate, called tetraborate, was reported to exist in three forms, which were identified by their nonindexed

[^0]powder patterns [16]; only the $\alpha$ form, whose structure is known [10], is characterized by an indexed powder pattern [17]. However, a more recent study [18] has led to four other forms of tetraborate that were all identified by indexed X-ray diffraction powder patterns. The aim of this work was to obtain single crystals of some forms of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ to solve their structures, and compare them with those of other $M_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ ( $M$ monovalent element) compounds described in a recent review [19].

## 2. Experimental section

### 2.1. Synthesis

Starting with borax, $\mathrm{Na}_{2}\left[\mathrm{~B}_{4} \mathrm{O}_{5}(\mathrm{OH})_{4}\right] \cdot 8 \mathrm{H}_{2} \mathrm{O}$ (analytical reagent grade, Prolabo), a dehydration was undertaken by heating until complete melting at $800^{\circ} \mathrm{C}$; then
the mixture was slowly cooled down to $750^{\circ} \mathrm{C}$ at a rate of $0.05^{\circ} \mathrm{C} / \mathrm{min}$, and annealed at this temperature for 10 h . This process, lowering the temperature by $50^{\circ} \mathrm{C}$ and annealing for 10 h , was repeated five times until the sample temperature reached $500^{\circ} \mathrm{C}$. Later, the furnace was turned off, and the mixture left to cool down to room temperature.

### 2.2. Single-crystal structure determination

The preparation was not very homogeneous: some fragments were noncrystalline (glass), others were polycrystalline, but all of the tested single crystals gave the same orthorhombic unit cell $(a=33.36 \AA$, $b=9.554 \AA, c=10.643 \AA$ ).

The intensity data were collected on a Bruker AXS SMART three-circle diffractometer using graphite monochromatized MoK $\alpha$ radiation $(\lambda=0.71073 \AA)$, and equipped with a CCD two-dimensional detector. Absorption corrections were made using the semiempirical SADABS program [20].

Two space groups were compatible with the extinctions observed: Ibam or Iba2. Resolution attempts by direct methods in the centrosymmetric group did not yield any recognizable solution. Conversely, in $I b a 2$ group, the resolution proceeded smoothly to $R_{1}=$ 0.0385 and $R_{\mathrm{w}}=0.0966$ with a formula $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$, not very far from the expected composition $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$. The Flack parameter [21] had a value of 0.13 ( 0.33 ), so the absolute configuration of the measured crystal was probably correct. With another crystal of the same preparation, the refinement leads to all of the atoms with inverse coordinates.

Considering the content of the cell (48 sodium, 104 boron and 180 oxygen atoms), it is quite evident that using the Ibam group (multiplicity of general position 16) would force to put at least one oxygen atom on a site with $2 / m$ or 222 symmetry (multiplicity of position 4 ), which is quite improbable.

All of the atoms are in general positions in $I b a 2$ space group, except for $\mathrm{O}(23)$ that lies on a twofold axis, in $4 a$ Wyckoff site. Crystal data and conditions of intensity collection are given in Table 1. The structure was refined by full-matrix least-squares techniques using the SHELXTL crystallographic software package [22]. The atomic coordinates and the displacement parameters are reported in Tables 2 and 3; significant bond lengths and angles are listed in Table 4.

## 3. Description of the crystal structure and discussion

### 3.1. Borate anion

The borate anion found in $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ is formed by a complex association of $\mathrm{BO}_{3}$ triangles ( $\Delta$ ) and $\mathrm{BO}_{4}$

Table 1
Crystal data and structure refinement for $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$

| Formula weight $(\mathrm{g} / \mathrm{mol})$ | 638.47 |
| :--- | :--- |
| Temperature $(\mathrm{K})$ | $293(2)$ |
| Wavelength $(\AA)$ | 0.71073 |
| Crystal system | Orthorhombic |
| Space group | $I b a 2($ No. 45$)$ |
| Unit cell dimensions $(\AA)$ | $a=33.359(11)$ |
|  | $b=9.554(3)$ |
|  | $c=10.644(4)$ |
| Volume $\left(\AA^{3}\right)$ | $3392.4(19)$ |
| $Z$ | 8 |
| Density, $D x\left(\mathrm{~g} / \mathrm{cm}^{3}\right)$ | 2.500 |
| Absorption coefficient $\left(\mathrm{mm}^{-1}\right)$ | 0.357 |
| $F$ (000) | 2488 |
| Crystal size $\left(\mathrm{mm}^{3}\right)$ | $0.45 \times 0.28 \times 0.15$ |
| $\theta$ range for data collection $(\mathrm{deg})$ | $2.22-28.05$ |
| Index ranges | $-43 \leqslant h \leqslant 41$ |
|  | $-12 \leqslant k \leqslant 12$ |
|  | $-13 \leqslant l \leqslant 13$ |
| Reflections collected | 12663 |
| Independent reflections | 3733 |
| Independent reflections $[I>2 \sigma(I)]$ | 3226 |
| $R_{\text {int }}$ | 0.0362 |
| Refinement method | $\mathrm{Full-matrix} \mathrm{least-squares} \mathrm{on} F^{2}$ |
| Number of variables | 375 |
| Goodness-of-fit on $F^{2}$ | 1.095 |
| Final $R$ indices $[I>2 \sigma(I)]$ | $R_{1}=0.0385$ |
| Final $R$ indices $($ all data $)$ | $\mathrm{w} R_{2}=0.0925$ |
| Largest difference map | $R_{1}=0.0456$ |
| peak and hole $\left(\mathrm{e} / \AA^{3}\right)$ | $\mathrm{w} R_{2}=0.0966$ |
|  |  |

tetrahedra (T) linked together by the vertices. All of the oxygen atoms are common to two triangles, two tetrahedra, or to one triangle and one tetrahedron. The $\mathrm{B}-\mathrm{O}$ distances in this borate anion are comparable with those existing in other borates with shorter distances in $\mathrm{BO}_{3}$ triangles (from 1.338 to $1.400 \AA$ ) than in $\mathrm{BO}_{4}$ tetrahedra (from 1.420 to $1.517 \AA$ ) (Table 4). A bond valence analysis [23] of the structure of $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ is performed, and included in Table 4; it is interesting to note that the values of the valence sums corresponding to boron atoms are around 3, ranging from 2.958 for $\mathrm{B}(13)$ to 3.052 for $\mathrm{B}(1)$.

The structure of this sodium borate can be described from a three-dimensional association of new complex anions with 13 boron atoms; it is formed by two linked tetraborate groups $\left(\mathrm{B}_{4} \mathrm{O}_{9}\right)$ bonded to a pentaborate group ( $\mathrm{B}_{5} \mathrm{O}_{11}$ ) (Fig. 1). These tetraborate and pentaborate groups were often met in complex borates as reported in a recent review [19]. The first tetraborate group is formed of two $\mathrm{BO}_{3}$ triangles $(\mathrm{B}(1)$ and $\mathrm{B}(2)$ ) and two $\mathrm{BO}_{4}$ tetrahedra $(\mathrm{B}(3)$ and $\mathrm{B}(4)$ ); the second tetraborate group also contains two $\mathrm{BO}_{3}$ triangles $(\mathrm{B}(5)$ and $\mathrm{B}(8)$ ) and two $\mathrm{BO}_{4}$ tetrahedra $(\mathrm{B}(6)$ and $\mathrm{B}(7))$; the pentaborate group consists of three $\mathrm{BO}_{3}$ triangles $(\mathrm{B}(10$, $\mathrm{B}(12)$ and $\mathrm{B}(13))$ and two $\mathrm{BO}_{4}$ tetrahedra $(\mathrm{B}(9)$ and

Table 2
Atomic coordinates $\left(\times 10^{5}\right)$ and equivalent isotropic displacement parameters $\left(\AA^{2} \times 10^{4}\right)$ for $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ with e.s.d.'s in parentheses

| Atoms | $x$ | $y$ | $z$ | $U_{\text {eq }}{ }^{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}(1)$ | 54509(5) | 93450(13) | 14581(15) | 283(4) |
| $\mathrm{Na}(2)$ | 69979(4) | 60529(13) | 254(13) | 241(3) |
| $\mathrm{Na}(3)$ | 54450(4) | 142049(14) | -26660(14) | 290(4) |
| $\mathrm{Na}(4)$ | 62350(4) | 34421(14) | 5176(14) | 275(3) |
| $\mathrm{Na}(5)$ | 70318(4) | 111348(14) | -13085(13) | 260(3) |
| $\mathrm{Na}(6)$ | 62768(4) | 87666(15) | 82859(14) | 318(3) |
| $\mathrm{O}(1)$ | 71411(6) | 42260(20) | 12480(20) | 166(5) |
| $\mathrm{O}(2)$ | 72755(6) | 19160(20) | 5250(20) | 149(4) |
| $\mathrm{O}(3)$ | 74698(7) | 39464(18) | -6490(30) | 189(4) |
| $\mathrm{O}(4)$ | 72595(6) | 30060(20) | 32270(20) | 161(5) |
| $\mathrm{O}(5)$ | 71862(6) | 6780(20) | 24840(20) | 158(4) |
| $\mathrm{O}(6)$ | 67026(5) | 23810(20) | 18710(20) | 151(4) |
| $\mathrm{O}(7)$ | 67664(6) | 1870(20) | 6860(20) | 174(5) |
| $\mathrm{O}(8)$ | 67182(6) | 45740(20) | 30580(20) | 177(5) |
| $\mathrm{O}(9)$ | 64179(7) | 80630(20) | 3840(30) | 186(5) |
| $\mathrm{O}(10)$ | 64487(6) | 91090(20) | 24310(20) | 179(5) |
| $\mathrm{O}(11)$ | 59130(5) | 75180(20) | 19380(30) | 148(4) |
| $\mathrm{O}(12)$ | 64364(7) | 68010(20) | 34000(20) | 190(5) |
| $\mathrm{O}(13)$ | 59742(7) | 86140(20) | 40020(20) | 201(5) |
| O(14) | 63936(6) | 57550(20) | 13670(20) | 175(5) |
| O(15) | 58950(7) | 63760(20) | -740(20) | 200(5) |
| $\mathrm{O}(16)$ | 56703(7) | 82810(20) | 59940(20) | 200(5) |
| $\mathrm{O}(17)$ | 54887(6) | 67140(20) | 43030(20) | 191(5) |
| $\mathrm{O}(18)$ | 52766(7) | 90860(20) | 42090(30) | 291(6) |
| $\mathrm{O}(19)$ | 57029(7) | 69140(20) | 78700(20) | 202(5) |
| $\mathrm{O}(20)$ | 53435(7) | 79150(20) | 95750(20) | 223(5) |
| $\mathrm{O}(21)$ | 52130(7) | 63060(20) | 63330(20) | 184(5) |
| $\mathrm{O}(22)$ | 51326(7) | 84510(20) | 74800(20) | 215(5) |
| $\mathrm{O}(23)$ | 50000 | 50000 | 45280(30) | 195(6) |
| B(1) | 72979(10) | 33380(40) | 3910(30) | 147(7) |
| B(2) | 73240(10) | 16100(30) | 33370(40) | 152(7) |
| B(3) | 69697(10) | 12800(40) | 14120(40) | 152(7) |
| B(4) | 69439(10) | 35530(40) | 23320(40) | 147(7) |
| B(5) | 65404(10) | 91320(40) | 11640(40) | 168(7) |
| B(6) | 61729(10) | 79980(40) | 29570(50) | 140(7) |
| B(7) | 61446(11) | 69480(40) | 9170(30) | 158(7) |
| B(8) | 65136(10) | 57070(40) | 26090(30) | 149(7) |
| B(9) | 56186(10) | 81910(30) | 46300(30) | 158(7) |
| B(10) | 56467(11) | 70480(30) | 91260(40) | 163(7) |
| B(11) | 54361(8) | 75220(40) | 68890(40) | 150(6) |
| B(12) | 52397(11) | 60360(40) | 50790(40) | 196(7) |
| B(13) | 50684(11) | 85350(40) | 87530(30) | 205(8) |

${ }^{\mathrm{a}} U_{\mathrm{eq}}$ is defined as one-third of the trace of the orthogonalized $U_{i j}$ tensor.

B(11)) (Fig. 2). Each borate anion with 13 boron atoms is linked to nine other similar groups to form a threedimensional network (Fig. 3). Following the crystal chemical classification of borates proposed by Christ and Clark [24], Heller [25], and slightly modified [19], the shorthand notation of this new complex association of $\mathrm{BO}_{3}$ triangles $(\Delta)$ and $\mathrm{BO}_{4}$ tetrahedra $(\mathrm{T})$ can be written with the fundamental building block (FBB): 13: $\infty^{3}$ [(5: $3 \Delta+2 \mathrm{~T})+2(4: 2 \Delta+2 \mathrm{~T})]$. It is interesting to note that this new FBB with 13 boron atoms completes the list of FBBs described in the recent review concerning crystal chemistry of alkaline and pseudo-alkaline

Table 3
Anisotropic displacement parameters $\left(\AA^{2} \times 10^{4}\right)$ (e.s.d.'s in parentheses) for all of the atoms of $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$

|  | $U_{11}$ | $U_{22}$ | $U_{33}$ | $U_{23}$ | $U_{13}$ | $U_{12}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Na}(1)$ | $256(7)$ | $209(7)$ | $383(10)$ | $-32(6)$ | $51(6)$ | $2(5)$ |
| $\mathrm{Na}(2)$ | $306(7)$ | $235(7)$ | $182(7)$ | $60(6)$ | $18(6)$ | $-14(6)$ |
| $\mathrm{Na}(3)$ | $323(8)$ | $298(7)$ | $249(9)$ | $57(6)$ | $10(6)$ | $115(6)$ |
| $\mathrm{Na}(4)$ | $243(7)$ | $351(8)$ | $230(8)$ | $-37(6)$ | $-29(6)$ | $-10(6)$ |
| $\mathrm{Na}(5)$ | $303(7)$ | $253(7)$ | $224(8)$ | $-75(6)$ | $-27(6)$ | $-19(6)$ |
| $\mathrm{Na}(6)$ | $330(8)$ | $333(8)$ | $292(9)$ | $72(6)$ | $95(7)$ | $6(6)$ |
| $\mathrm{O}(1)$ | $192(10)$ | $151(10)$ | $155(12)$ | $4(9)$ | $24(10)$ | $4(8)$ |
| $\mathrm{O}(2)$ | $161(10)$ | $136(10)$ | $150(12)$ | $7(8)$ | $11(9)$ | $-15(8)$ |
| $\mathrm{O}(3)$ | $263(11)$ | $137(9)$ | $167(9)$ | $3(10)$ | $83(8)$ | $1(10)$ |
| $\mathrm{O}(4)$ | $182(11)$ | $145(10)$ | $155(12)$ | $-11(9)$ | $-26(10)$ | $9(8)$ |
| $\mathrm{O}(5)$ | $217(11)$ | $150(10)$ | $107(11)$ | $2(8)$ | $-21(9)$ | $-2(8)$ |
| $\mathrm{O}(6)$ | $144(9)$ | $159(9)$ | $149(9)$ | $1(7)$ | $7(10)$ | $-5(9)$ |
| $\mathrm{O}(7)$ | $219(10)$ | $184(11)$ | $119(11)$ | $-1(8)$ | $1(9)$ | $-74(9)$ |
| $\mathrm{O}(8)$ | $218(11)$ | $167(11)$ | $146(11)$ | $7(9)$ | $3(9)$ | $65(8)$ |
| $\mathrm{O}(9)$ | $243(12)$ | $182(11)$ | $134(12)$ | $1(8)$ | $19(9)$ | $-71(9)$ |
| $\mathrm{O}(10)$ | $200(11)$ | $206(11)$ | $130(12)$ | $-6(9)$ | $1(9)$ | $-69(9)$ |
| $\mathrm{O}(11)$ | $137(9)$ | $148(9)$ | $159(9)$ | $-5(7)$ | $-10(10)$ | $4(9)$ |
| $\mathrm{O}(12)$ | $249(12)$ | $170(11)$ | $152(12)$ | $-28(9)$ | $-49(10)$ | $56(9)$ |
| $\mathrm{O}(13)$ | $205(11)$ | $206(12)$ | $191(13)$ | $-30(9)$ | $60(9)$ | $-31(9)$ |
| $\mathrm{O}(14)$ | $217(11)$ | $174(11)$ | $135(12)$ | $-30(9)$ | $-28(9)$ | $49(9)$ |
| $\mathrm{O}(15)$ | $230(12)$ | $171(11)$ | $199(12)$ | $-48(9)$ | $-89(10)$ | $31(9)$ |
| $\mathrm{O}(16)$ | $279(12)$ | $180(11)$ | $140(12)$ | $1(8)$ | $6(9)$ | $-64(9)$ |
| $\mathrm{O}(17)$ | $248(11)$ | $166(10)$ | $159(11)$ | $-40(8)$ | $41(9)$ | $-41(9)$ |
| $\mathrm{O}(18)$ | $220(11)$ | $196(12)$ | $458(15)$ | $26(11)$ | $-133(11)$ | $-5(9)$ |
| $\mathrm{O}(19)$ | $195(11)$ | $271(12)$ | $139(12)$ | $-4(9)$ | $-13(9)$ | $64(9)$ |
| $\mathrm{O}(20)$ | $264(12)$ | $264(13)$ | $140(11)$ | $-13(9)$ | $6(9)$ | $92(10)$ |
| $\mathrm{O}(21)$ | $237(11)$ | $174(10)$ | $143(12)$ | $-1(8)$ | $18(9)$ | $-33(9)$ |
| $\mathrm{O}(22)$ | $243(12)$ | $193(11)$ | $207(13)$ | $-28(9)$ | $-16(10)$ | $69(9)$ |
| $\mathrm{O}(23)$ | $228(14)$ | $199(14)$ | $158(15)$ | 0 | 0 | $-42(13)$ |
| $\mathrm{B}(1)$ | $140(15)$ | $158(16)$ | $144(19)$ | $6(14)$ | $1(14)$ | $0(13)$ |
| $\mathrm{B}(2)$ | $149(16)$ | $149(15)$ | $159(19)$ | $24(14)$ | $19(14)$ | $-15(13)$ |
| $\mathrm{B}(3)$ | $132(16)$ | $171(17)$ | $150(20)$ | $39(13)$ | $-3(14)$ | $-28(12)$ |
| $\mathrm{B}(4)$ | $141(15)$ | $163(17)$ | $136(19)$ | $-19(13)$ | $23(14)$ | $23(13)$ |
| $\mathrm{B}(5)$ | $160(16)$ | $172(17)$ | $173(19)$ | $0(13)$ | $8(15)$ | $-6(13)$ |
| $\mathrm{B}(6)$ | $132(16)$ | $122(16)$ | $167(17)$ | $26(12)$ | $11(13)$ | $-5(13)$ |
| $\mathrm{B}(7)$ | $171(17)$ | $160(17)$ | $143(18)$ | $28(12)$ | $-31(13)$ | $-20(13)$ |
| $\mathrm{B}(8)$ | $162(15)$ | $120(15)$ | $164(18)$ | $-32(13)$ | $19(15)$ | $-10(12)$ |
| $\mathrm{B}(9)$ | $161(16)$ | $150(17)$ | $162(18)$ | $6(13)$ | $-3(13)$ | $-5(13)$ |
| $\mathrm{B}(10)$ | $165(16)$ | $134(17)$ | $191(18)$ | $-23(13)$ | $-7(14)$ | $-25(13)$ |
| $\mathrm{B}(11)$ | $163(13)$ | $171(14)$ | $115(13)$ | $-19(11)$ | $-17(15)$ | $-11(15)$ |
| $\mathrm{B}(12)$ | $225(18)$ | $142(17)$ | $222(18)$ | $18(14)$ | $-19(16)$ | $-13(14)$ |
| $\mathrm{B}(13)$ | $188(18)$ | $181(17)$ | $246(19)$ | $24(14)$ | $-6(14)$ | $-33(14)$ |
| T |  |  |  |  |  |  |

The anisotropic displacement factor exponent takes the form: $-2 \pi^{2}\left(h^{2} a^{*} U_{11}+\cdots\right)$.
$\left(\mathrm{Ag}^{+}, \mathrm{Tl}^{+}, \mathrm{NH}_{4}^{+}\right)$borates [19], between those with 12 boron atoms, $12: 2[(6: 3 \Delta+3 T)]$ or $12:[(7: 4 \Delta+3 T)+(5:$ $2 \Delta+3 \mathrm{~T})$ ] and FBB with 14 boron atoms, 14: [2(5: $3 \Delta+2 T)+(3: 2 \Delta+T)+(1: T)]$. Note the good agreement with the calculation of the ratio of $\mathrm{BO}_{3}$ triangles to $\mathrm{BO}_{4}$ tetrahedra from the Parthé formula [26].

To our knowledge, only three other compounds with 13 boron atoms were reported. The first one is a new potassium manganese borate recently solvothermally synthesized [27]; its structural formula is $\mathrm{K}_{7}\left\{\left(\mathrm{BO}_{3}\right)\right.$ $\left.\mathrm{Mn}\left[\mathrm{B}_{12} \mathrm{O}_{18}(\mathrm{OH})_{6}\right]\right\} \cdot \mathrm{H}_{2} \mathrm{O}$ with only six and a half crystallographically independent boron atoms. The

Table 4
Interatomic distances $(\AA)$, angles $\left({ }^{\circ}\right)$, and bond valence analysis* in $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$

| $\mathrm{BO}_{3}$ triangles |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $s$ |  |  |
| $\mathrm{B}(1)-\mathrm{O}(1)$ | 1.351(4) | 1.055 | $\mathrm{O}(1)-\mathrm{B}(1)-\mathrm{O}(2)$ | 122.1(3) |
| $\mathrm{B}(1)-\mathrm{O}(2)$ | 1.368(4) | 1.008 | $\mathrm{O}(1)-\mathrm{B}(1)-\mathrm{O}(3)$ | 116.0(3) |
| $\mathrm{B}(1)-\mathrm{O}(3)$ | 1.375(4) | 0.989 | $\mathrm{O}(2)-\mathrm{B}(1)-\mathrm{O}(3)$ | 121.8(3) |
| Mean | $1.365(4)$ | $\Sigma s=3.052$ | Mean | 120.0(3) |
| $\mathrm{B}(2)-\mathrm{O}(5)$ | 1.353(4) | 1.050 | $\mathrm{O}(5)-\mathrm{B}(2)-\mathrm{O}(4)$ | 122.4(3) |
| $\mathrm{B}(2)-\mathrm{O}(4)$ | 1.356(4) | 1.041 | $\mathrm{O}(5)-\mathrm{B}(2)-\mathrm{O}(3)^{\mathrm{a}}$ | 116.0(3) |
| $\mathrm{B}(2)-\mathrm{O}(3)^{\mathrm{a}}$ | 1.386(4) | 0.960 | $\mathrm{O}(4)-\mathrm{B}(2)-\mathrm{O}(3)^{\mathrm{a}}$ | 121.5(3) |
| Mean | $1.365(4)$ | $\Sigma s=3.051$ | Mean | 120.0(3) |
| $\mathrm{B}(5)-\mathrm{O}(7)^{\mathrm{b}}$ | 1.358(4) | 1.036 | $\mathrm{O}(7)^{\mathrm{b}}-\mathrm{B}(5)-\mathrm{O}(9)$ | 119.3(3) |
| $\mathrm{B}(5)-\mathrm{O}(9)$ | 1.378(4) | 0.981 | $\mathrm{O}(7)^{\mathrm{b}}-\mathrm{B}(5)-\mathrm{O}(10)$ | 120.0(3) |
| $\mathrm{B}(5)-\mathrm{O}(10)$ | 1.382(4) | 0.971 | $\mathrm{O}(9)-\mathrm{B}(5)-\mathrm{O}(10)$ | 120.6(3) |
| Mean | 1.373(4) | $\Sigma s=2.988$ | Mean | 120.0(3) |
| $\mathrm{B}(8)-\mathrm{O}(8)$ | 1.366(4) | 1.014 | $\mathrm{O}(8)-\mathrm{B}(8)-\mathrm{O}(12)$ | 119.0(3) |
| $\mathrm{B}(8)-\mathrm{O}(12)$ | 1.367(4) | 1.011 | $\mathrm{O}(8)-\mathrm{B}(8)-\mathrm{O}(14)$ | 120.4(3) |
| $\mathrm{B}(8)-\mathrm{O}(14)$ | 1.382(4) | 0.971 | $\mathrm{O}(12)-\mathrm{B}(8)-\mathrm{O}(14)$ | 120.6(3) |
| Mean | 1.372(4) | $\Sigma s=2.996$ | Mean | 120.0(3) |
| $\mathrm{B}(10)-\mathrm{O}(15)^{\mathrm{c}}$ | 1.351(4) | 1.055 | $\mathrm{O}(15)^{\mathrm{c}}-\mathrm{B}(10)-\mathrm{O}(19)$ | 119.4(3) |
| $\mathrm{B}(10)-\mathrm{O}(19)$ | $1.356(5)$ | 1.041 | $\mathrm{O}(15)^{\mathrm{c}}-\mathrm{B}(10)-\mathrm{O}(20)$ | 120.8(3) |
| $\mathrm{B}(10)-\mathrm{O}(20)$ | 1.392(4) | 0.945 | $\mathrm{O}(19)-\mathrm{B}(10)-\mathrm{O}(20)$ | 119.7(3) |
| Mean | 1.366(4) | $\Sigma s=3.041$ | Mean | 119.9(3) |
| $\mathrm{B}(12)-\mathrm{O}(17)$ | 1.338(4) | 1.093 | $\mathrm{O}(17)-\mathrm{B}(12)-\mathrm{O}(21)$ | 123.6(3) |
| $\mathrm{B}(12)-\mathrm{O}(21)$ | 1.363(4) | 1.022 | $\mathrm{O}(17)-\mathrm{B}(12)-\mathrm{O}(23)$ | 116.0(3) |
| $\mathrm{B}(12)-\mathrm{O}(23)$ | 1.401(4) | 0.922 | $\mathrm{O}(21)-\mathrm{B}(12)-\mathrm{O}(23)$ | 120.4(3) |
| Mean | 1.367(4) | $\Sigma s=3.037$ | Mean | 120.0(3) |
| $\mathrm{B}(13)-\mathrm{O}(18)^{\text {d }}$ | 1.356(4) | 1.041 | $\mathrm{O}(18)^{\mathrm{d}}-\mathrm{B}(13)-\mathrm{O}(22)$ | 120.6(3) |
| $\mathrm{B}(13)-\mathrm{O}(22)$ | 1.374(4) | 0.992 | $\mathrm{O}(18)^{\mathrm{d}}-\mathrm{B}(13)-\mathrm{O}(20)$ | 119.8(3) |
| $\mathrm{B}(13)-\mathrm{O}(20)$ | 1.400(4) | 0.925 | $\mathrm{O}(22)-\mathrm{B}(13)-\mathrm{O}(20)$ | 119.3(3) |
| Mean | 1.377(4) | $\Sigma s=2.958$ | Mean | 119.9(3) |
| $\mathrm{BO}_{4}$ tetrahedra |  |  |  |  |
|  |  | $s$ |  |  |
| $\mathrm{B}(3)-\mathrm{O}(6)$ | 1.463(4) | 0.780 | $\mathrm{O}(6)-\mathrm{B}(3)-\mathrm{O}(7)$ | 114.0(2) |
| $\mathrm{B}(3)-\mathrm{O}(7)$ | 1.465(4) | 0.776 | $\mathrm{O}(6)-\mathrm{B}(3)-\mathrm{O}(5)$ | 108.8(3) |
| $\mathrm{B}(3)-\mathrm{O}(5)$ | 1.467(4) | 0.771 | $\mathrm{O}(6)-\mathrm{B}(3)-\mathrm{O}(2)$ | 109.2(2) |
| $\mathrm{B}(3)-\mathrm{O}(2)$ | 1.517(4) | 0.674 | $\mathrm{O}(7)-\mathrm{B}(3)-\mathrm{O}(5)$ | 111.0(3) |
| Mean | 1.478(4) | $\Sigma s=3.001$ | $\mathrm{O}(7)-\mathrm{B}(3)-\mathrm{O}(2)$ | 105.6(3) |
|  |  |  | $\mathrm{O}(5)-\mathrm{B}(3)-\mathrm{O}(2)$ | 108.1(2) |
|  |  |  | Mean | 109.4(3) |
| $\mathrm{B}(4)-\mathrm{O}(8)$ | 1.454(4) | 0.799 | $\mathrm{O}(8)-\mathrm{B}(4)-\mathrm{O}(6)$ | 114.0(2) |
| $\mathrm{B}(4)-\mathrm{O}(6)$ | 1.464(4) | 0.778 | $\mathrm{O}(8)-\mathrm{B}(4)-\mathrm{O}(1)$ | 110.7(3) |
| $\mathrm{B}(4)-\mathrm{O}(1)$ | 1.476(4) | 0.753 | $\mathrm{O}(8)-\mathrm{B}(4)-\mathrm{O}(4)$ | 104.9(3) |
| $\mathrm{B}(4)-\mathrm{O}(4)$ | 1.513(4) | 0.681 | $\mathrm{O}(6)-\mathrm{B}(4)-\mathrm{O}(1)$ | 108.5(3) |
| Mean | 1.477(4) | $\Sigma s=3.011$ | $\mathrm{O}(6)-\mathrm{B}(4)-\mathrm{O}(4)$ | 109.2(3) |
|  |  |  | $\mathrm{O}(1)-\mathrm{B}(4)-\mathrm{O}(4)$ | 109.4(2) |
|  |  |  | Mean | 109.4(3) |
| $\mathrm{B}(6)-\mathrm{O}(13)$ | 1.422(4) | 0.871 | $\mathrm{O}(13)-\mathrm{B}(6)-\mathrm{O}(11)$ | 115.7(3) |
| $\mathrm{B}(6)-\mathrm{O}(11)$ | 1.462(4) | 0.782 | $\mathrm{O}(13)-\mathrm{B}(6)-\mathrm{O}(10)$ | 106.4(3) |
| $\mathrm{B}(6)-\mathrm{O}(10)$ | 1.512(4) | 0.683 | $\mathrm{O}(13)-\mathrm{B}(6)-\mathrm{O}(12)$ | 109.9(3) |
| $\mathrm{B}(6)-\mathrm{O}(12)$ | 1.517(4) | 0.674 | $\mathrm{O}(11)-\mathrm{B}(6)-\mathrm{O}(10)$ | 107.8(3) |
| Mean | 1.478(4) | $\Sigma s=3.010$ | $\mathrm{O}(11)-\mathrm{B}(6)-\mathrm{O}(12)$ | 109.7(3) |
|  |  |  | $\mathrm{O}(10)-\mathrm{B}(6)-\mathrm{O}(12)$ | 106.9(2) |
|  |  |  | Mean | 109.4(3) |
| $\mathrm{B}(7)-\mathrm{O}(11)$ | 1.440(4) | 0.830 | $\mathrm{O}(11)-\mathrm{B}(7)-\mathrm{O}(15)$ | 112.5(3) |
| $\mathrm{B}(7)-\mathrm{O}(15)$ | 1.451(4) | 0.806 | $\mathrm{O}(11)-\mathrm{B}(7)-\mathrm{O}(14)$ | 110.2(3) |
| $\mathrm{B}(7)-\mathrm{O}(14)$ | 1.490(4) | 0.725 | $\mathrm{O}(11)-\mathrm{B}(7)-\mathrm{O}(9)$ | 109.9(3) |
| $\mathrm{B}(7)-\mathrm{O}(9)$ | 1.513(4) | 0.681 | $\mathrm{O}(15)-\mathrm{B}(7)-\mathrm{O}(14)$ | 105.4(3) |
| Mean | 1.474(4) | $\Sigma s=3.042$ | $\mathrm{O}(15)-\mathrm{B}(7)-\mathrm{O}(9)$ | 109.8(3) |
|  |  |  | $\mathrm{O}(14)-\mathrm{B}(7)-\mathrm{O}(9)$ | 108.9(3) |
|  |  |  | Mean | 109.4(3) |

Table 4 (continued)

| $\mathrm{B}(9)-\mathrm{O}(13)$ | $1.420(4)$ | 0.876 | $\mathrm{O}(13)-\mathrm{B}(9)-\mathrm{O}(16)$ | $110.6(3)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{B}(9)-\mathrm{O}(16)$ | $1.464(4)$ | 0.778 | $\mathrm{O}(13)-\mathrm{B}(9)-\mathrm{O}(18)$ | $109.5(3)$ |
| $\mathrm{B}(9)-\mathrm{O}(18)$ | $1.494(4)$ | 0.717 | $\mathrm{O}(13)-\mathrm{B}(9)-\mathrm{O}(17)$ | $113.3(3)$ |
| $\mathrm{B}(9)-\mathrm{O}(17)$ | $1.517(4)$ | 0.674 | $\mathrm{O}(16)-\mathrm{B}(9)-\mathrm{O}(18)$ | $110.7(3)$ |
| Mean | $1.474(4)$ | $\Sigma s=3.045$ | $\mathrm{O}(16)-\mathrm{B}(9)-\mathrm{O}(17)$ | $108.4(3)$ |
|  |  |  | $\mathrm{O}(18)-\mathrm{B}(9)-\mathrm{O}(17)$ | $104.2(3)$ |
|  |  | Mean | $109.4(3)$ |  |
| $\mathrm{B}(11)-\mathrm{O}(16)$ | $1.430(5)$ | 0.853 | $\mathrm{O}(16)-\mathrm{B}(11)-\mathrm{O}(22)$ | $110.6(3)$ |
| $\mathrm{B}(11)-\mathrm{O}(22)$ | $1.486(4)$ | 0.733 | $\mathrm{O}(16)-\mathrm{B}(11)-\mathrm{O}(19)$ | $109.8(2)$ |
| $\mathrm{B}(11)-\mathrm{O}(19)$ | $1.490(4)$ | 0.725 | $\mathrm{O}(16)-\mathrm{B}(11)-\mathrm{O}(21)$ | $113.6(3)$ |
| $\mathrm{B}(11)-\mathrm{O}(21)$ | $1.501(4)$ | 0.704 | $\mathrm{O}(22)-\mathrm{B}(11)-\mathrm{O}(19)$ | $110.1(3)$ |
| Mean | $1.477(4)$ | $\Sigma s=3.015$ | $\mathrm{O}(22)-\mathrm{B}(11)-\mathrm{O}(21)$ | $106.9(2)$ |
|  |  |  | $\mathrm{O}(19)-\mathrm{B}(11)-\mathrm{O}(21)$ | $105.7(3)$ |
|  |  | Mean | $109.4(3)$ |  |

Environment of sodium atoms $(<3.00$ A $)$

|  |  | $s$ |  |  | $s$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}(1)-\mathrm{O}(11)$ | 2.384(2) | 0.208 | $\mathrm{Na}(4)-\mathrm{O}(6)$ | 2.353(3) | 0.226 |
| $\mathrm{Na}(1)-\mathrm{O}(22)^{\text {e }}$ | 2.388(3) | 0.206 | $\mathrm{Na}(4)-\mathrm{O}(12)^{\mathrm{h}}$ | 2.364(3) | 0.220 |
| $\mathrm{Na}(1)-\mathrm{O}(16)^{\text {f }}$ | 2.434(3) | 0.182 | $\mathrm{Na}(4)-\mathrm{O}(14)$ | 2.445 (3) | 0.176 |
| $\mathrm{Na}(1)-\mathrm{O}(20)^{\mathrm{g}}$ | 2.452(3) | 0.173 | $\mathrm{Na}(4)-\mathrm{O}(16)^{\mathrm{h}}$ | 2.553(3) | 0.132 |
| $\mathrm{Na}(1)-\mathrm{O}(22)^{\text {f }}$ | 2.597(3) | 0.117 | $\mathrm{Na}(4)-\mathrm{O}(13)^{\mathrm{h}}$ | 2.687(3) | 0.092 |
| $\mathrm{Na}(1)-\mathrm{O}(18)^{\mathrm{f}}$ | 2.883(3) | 0.054 |  |  | $\Sigma s=0.846$ |
| $\mathrm{Na}(1)-\mathrm{O}(18)$ | 2.996 (3) | 0.040 | $\mathrm{Na}(4)-\mathrm{B}(9)^{\mathrm{h}}$ | 2.749(4) |  |
|  |  | $\Sigma s=0.980$ | $\mathrm{Na}(4)-\mathrm{O}(17)^{\mathrm{h}}$ | 2.809(3) |  |
| $\mathrm{Na}(2)-\mathrm{O}(1)$ | 2.229(3) | 0.316 | $\mathrm{Na}(5)-\mathrm{O}(5)^{\mathrm{h}}$ | 2.217(3) | 0.327 |
| $\mathrm{Na}(2)-\mathrm{O}(4)^{\mathrm{h}}$ | 2.288(3) | 0.270 | $\mathrm{Na}(5)-\mathrm{O}(2)^{\text {b }}$ | 2.242(3) | 0.305 |
| $\mathrm{Na}(2)-\mathrm{O}(8)^{\mathrm{h}}$ | 2.370 (3) | 0.216 | $\mathrm{Na}(5)-\mathrm{O}(10)^{\text {f }}$ | 2.375 (3) | 0.213 |
| $\mathrm{Na}(2)-\mathrm{O}(14)$ | 2.487(3) | 0.157 | $\mathrm{Na}(5)-\mathrm{O}(7)^{\text {b }}$ | 2.472(3) | 0.164 |
| $\mathrm{Na}(2)-\mathrm{O}(2)^{\text {i }}$ | 2.615(3) | 0.111 | $\mathrm{Na}(5)-\mathrm{O}(4)^{1}$ | 2.551(3) | 0.132 |
| $\mathrm{Na}(2)-\mathrm{O}(3)$ | 2.654(2) | 0.100 | $\mathrm{Na}(5)-\mathrm{O}(3)^{\text {i }}$ | 2.762(2) | 0.075 |
| $\mathrm{Na}(2)-\mathrm{O}(9)$ | $2.753(3)$ | 0.077 | $\mathrm{Na}(5)-\mathrm{O}(12)^{\mathrm{f}}$ | 2.816(3) | 0.065 |
|  |  | $\Sigma s=1.247$ |  |  | $\Sigma s=1.281$ |
| $\mathrm{Na}(2)-\mathrm{B}(1)$ | 2.808(4) |  | $\mathrm{Na}(5)-\mathrm{B}(2){ }^{\text {h }}$ | 2.823(4) |  |
| $\mathrm{Na}(2)-\mathrm{B}(4)^{\mathrm{h}}$ | 2.897(4) |  | $\mathrm{Na}(5)-\mathrm{B}(3)^{\text {b }}$ | 2.907(4) |  |
|  |  |  | $\mathrm{Na}(5)-\mathrm{B}(1)^{\text {b }}$ | 2.914(4) |  |
| $\mathrm{Na}(3)-\mathrm{O}(17)^{\mathrm{f}}$ | 2.277(3) | 0.278 | $\mathrm{Na}(6)-\mathrm{O}(10)^{\mathrm{m}}$ | 2.298(3) | 0.262 |
| $\mathrm{Na}(3)-\mathrm{O}(11)^{\mathrm{f}}$ | 2.307(2) | 0.256 | $\mathrm{Na}(6)-\mathrm{O}(6)^{\mathrm{n}}$ | 2.343 (3) | 0.232 |
| $\mathrm{Na}(3)-\mathrm{O}(21)^{\mathrm{j}}$ | 2.401(3) | 0.199 | $\mathrm{Na}(6)-\mathrm{O}(9)^{\mathrm{c}}$ | 2.379(3) | 0.211 |
| $\mathrm{Na}(3)-\mathrm{O}(21)^{\mathrm{k}}$ | 2.488(3) | 0.157 | $\mathrm{Na}(6)-\mathrm{O}(19)$ | 2.644(3) | 0.103 |
| $\mathrm{Na}(3)-\mathrm{O}(19)^{\mathrm{j}}$ | 2.787(3) | 0.070 | $\mathrm{Na}(6)-\mathrm{O}(13)^{\mathrm{m}}$ | 2.804(3) | 0.067 |
| $\mathrm{Na}(3)-\mathrm{O}(23)^{\text {f }}$ | 2.869(3) | 0.056 |  |  | $\Sigma s=0.875$ |
|  |  | $\Sigma s=1.016$ | $\mathrm{Na}(6)-\mathrm{B}(10)$ | 2.813(4) |  |
| $\mathrm{Na}(3)-\mathrm{B}(7)^{\text {f }}$ | 2.989(4) |  |  |  |  |

[^1]*The results refer to the equation $s=\exp \left[\left(r_{0}-r\right) / \mathrm{B}\right]$ with $r_{0}=1.371 \AA$ and $1.803 \AA$ for $\mathrm{B}-\mathrm{O}$ and $\mathrm{Na}-\mathrm{O}$ bonds, respectively, and $B=0.37$ [23].


Fig. 1. Representation of the $\left(\mathrm{B}_{13} \mathrm{O}_{27}\right)$ borate group existing in $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$. It consists of the association of one pentaborate group $\left(\mathrm{B}_{5} \mathrm{O}_{11}\right)$ and two tetraborate groups $\left(\mathrm{B}_{4} \mathrm{O}_{9}\right)$; the linkage is made by O 13 and O 8 oxygen atoms. Atoms are represented by $50 \%$ ellipsoids.


Fig. 2. Representation of $\left(\mathrm{B}_{13} \mathrm{O}_{27}\right)$ borate group existing in $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5} . \mathrm{BO}_{3}$ triangles are in light and $\mathrm{BO}_{4}$ tetrahedra in dark. Only the boron atoms are labelled.
shorthand notation of the corresponding FBB can be written 13: $2[6(3 \Delta+3 \mathrm{~T})+0.5(1: \Delta)]$ following the rules of the classification of borates [19,24,25]. Even if there is the same number of $\mathrm{BO}_{3}$ triangles (seven) and $\mathrm{BO}_{4}$ tetrahedra (six) as in the studied borate, its structure is completely different; it consists of a distorted $\mathrm{O}_{2} \mathrm{MnO}_{3}$ square pyramid embedded between $\left[\mathrm{B}_{12} \mathrm{O}_{18}(\mathrm{OH})_{6}\right]$ and $\mathrm{BO}_{3}$ units. The $\left[\mathrm{B}_{12} \mathrm{O}_{18}(\mathrm{OH})_{6}\right]$ cluster is identical to the one existing in a hydrated sodium borate $\mathrm{Na}_{8}\left[\mathrm{~B}_{12} \mathrm{O}_{20}(\mathrm{OH})_{4}\right]$ [28] with the FBB at the abovementioned 12 boron atoms [19]. Another borate contains 13 boron atoms; this is a mineral named walkerite, the structure of which was recently solved;
this is a complex hydrated chloro-borate with the formula: $\left.\mathrm{Ca}_{16} \mathrm{Mg}^{2} \mathrm{~B}_{13} \mathrm{O}_{17}(\mathrm{OH})_{12}\right]_{4} \mathrm{Cl}_{6}\left(\mathrm{H}_{2} \mathrm{O}\right)_{28}$ [29]. The structure contains 13 crystallographically independent boron atoms with six $\mathrm{BO}_{3}$ triangles and seven $\mathrm{BO}_{4}$ tetrahedra, different from the studied borate. Another borate with 13 boron atoms is reported in a recent review [30] but it contains $12 \mathrm{BO}_{3}$ triangles and one $\mathrm{BO}_{4}$ tetrahedron. Therefore, the FBB found in $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ really corresponds to a new association of $\mathrm{BO}_{3}$ triangles and $\mathrm{BO}_{4}$ tetrahedra.

### 3.2. Sodium atom environments

The sodium atoms are located in six crystallographically independent positions. The $\mathrm{Na}-\mathrm{O}$ distances show a considerable variation, ranging from $2.217 \AA$ for $\mathrm{Na}(5)-\mathrm{O}(5)$ to $2.996 \AA$ for $\mathrm{Na}(1)-\mathrm{O}(18)$ (Table 4). These values are close to the ones found in the other anhydrous sodium borates of known structures (Table 5) [1,2,7-15]. However, some Na-B distances were inferior to the longest $\mathrm{Na}-\mathrm{O}$ bond, like $\mathrm{Na}(4)-\mathrm{B}(9), 2.749 \AA$. The bond valence analysis [23] of $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ structure cannot permit to obtain a more precise value of coordination number of the sodium atoms (Table 4); the values of the valence sums range from 0.846 for $\mathrm{Na}(4)$ to 1.281 for $\mathrm{Na}(5)$.
To complete the valence bond analysis (Table 4), it is interesting to note that the valence sum values concerning oxygen atoms are around 2 , from 1.852 for $\mathrm{O}(18)$ to 2.148 for $\mathrm{O}(5)$.

### 3.3. Phase transformations of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ and $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ formation

As for other hydrated alkaline tetraborates [31-34], the thermal decomposition of borax leads to an anhydrous amorphous phase [35]. By annealing this solid for different times and at different temperatures, nonindexed X-ray diffraction patterns were obtained, and attributed to three forms of sodium tetraborate [16]. In a more recent detailed study on "Dependence of crystallization behavior of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ on its glass structure and the characteristics of phase transformation" [18], four forms were obtained and identified by their indexed powder patterns; they all crystallize in monoclinic system, while the form with the known structure ([10,17], see Table 5) named $\alpha$, is triclinic. The following results were published [18], and the Smith and Snyder figures of merit [36] were calculated in Powder Diffraction File [37]: $\beta-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}: a=14.395 \AA$, $b=16.59 \AA, c=5.596 \AA, \beta=95.09^{\circ}, V=1331.13 \AA^{3}$, $Z=10, \quad F(30)=7(0.037, \quad 120) ; \quad \gamma-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}: \quad a=$ $18.311 \AA, \quad b=4.786 \AA, \quad c=12.37 \AA, \quad \beta=104.48^{\circ}$, $V=1049.63 \AA^{3}, \quad Z=8, \quad F(30)=6(0.044, \quad 117) ; \quad \delta-$ $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}: \quad a=12.636 \AA, \quad b=15.387 \AA, \quad c=5.812 \AA$, $\beta=106.98^{\circ}, \quad V=1080.77 \AA^{3}, Z=8, \quad F(30)=6(0.044$,


Fig. 3. Schematic representation of the content of the cell. Upper part: whole cell projected on $a b$ plane; lower part: contents of the slices $0.5<x<0.6,0.6<x<0.675$ and $0.675<x<0.75$, respectively. Black circles represent sodium atoms.

Table 5
Crystallographic and structural data of anhydrous sodium borates

| Formula | B/Na | Parameters ( $\AA$, deg); V $\left(\AA^{3}\right) ; Z$ | Space group | Borate anion | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}_{3} \mathrm{BO}_{3}$ | 0.33 | $\begin{aligned} & a=5.687 \\ & b=7.530 \\ & c=9.993 \\ & \beta=127.15 \\ & V=341.1 ; Z=4 \end{aligned}$ | $P 2_{1} / c$ | 1: [(1: $\Delta)]$ | [7] |
| $\mathrm{Na}_{4} \mathrm{~B}_{2} \mathrm{O}_{5}$ | 0.5 | $\begin{aligned} & a=10.618 \\ & b=8.015 \\ & c=6.287 \\ & \beta=110.1 \\ & V=502.46 ; Z=4 \end{aligned}$ | C2/c | 2: [(2: 2t) ] | [8] |
| $\mathrm{NaBO}_{2}\left(\mathrm{Na}_{3} \mathrm{~B}_{3} \mathrm{O}_{6}\right)$ | 1 | $\begin{aligned} & a=11.925 \\ & c=6.439 \\ & V=793 ; Z=6 \end{aligned}$ | $R-3 c$ | 3: $[(3: 3 \Delta)]$ | [9] |
| $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}\left(\mathrm{Na}_{4} \mathrm{~B}_{8} \mathrm{O}_{14}\right)$ | 2 | $\begin{aligned} & a=6.5445 \\ & b=8.6205 \\ & c=10.4855 \\ & \alpha=93.279 \\ & \beta=94.870 \\ & \gamma=90.843 \\ & V=588.3 ; Z=2 . \end{aligned}$ | $P-1$ | $8: \infty^{3}[(5: 3 \Delta+2 \mathrm{~T})+(3: 2 \Delta+\mathrm{T})]$ | [10] |
| $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ | 2.17 | $\begin{aligned} & a=33.359 \\ & b=9.554 \\ & c=10.644 \\ & V=3392.4 ; Z=8 \end{aligned}$ | Iba 2 | 13: $\infty^{3}[(5: 3 \Delta+2 \mathrm{~T})+2(4: 2 \Delta+2 \mathrm{~T})]$ | This work |
| $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}$ | 2.33 | $\begin{aligned} & a=6.638 \\ & b=8.249 \\ & c=8.836 \\ & \alpha=95.875 \end{aligned}$ | $P-1$ | 7: $\infty^{3}[(3: 2 \Delta+\mathrm{T})+(3: \Delta+2 \mathrm{~T})+(1: \Delta)]$ | [2] |

Table 5 (continued)

| Formula | B/Na | Parameters ( $\AA$, deg $) ; V\left(\AA^{3}\right) ; Z$ | Space group | Borate anion | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \beta=100.680 \\ & \gamma=99.688 \\ & V=464.2 ; Z=2 \end{aligned}$ |  |  |  |
| $\alpha-\mathrm{NaB}_{3} \mathrm{O}_{5}\left(\mathrm{Na}_{3} \mathrm{~B}_{9} \mathrm{O}_{15}\right)$ | 3 | $\begin{aligned} & a=10.085 \\ & b=11.363 \\ & c=10.845 \\ & \beta=104.48 \\ & V=1203.3 ; Z=2 \end{aligned}$ | $P 2{ }_{1} / c$ | 9: $\infty^{3}[(5: 4 \Delta+\mathrm{T})+(4: 2 \Delta+2 \mathrm{~T})]$ | [11] |
| $\beta-\mathrm{NaB}_{3} \mathrm{O}_{5}\left(\mathrm{Na}_{3} \mathrm{~B}_{9} \mathrm{O}_{15}\right)$ | 3 | $\begin{aligned} & a=8.990 \\ & b=11.033 \\ & c=12.107 \\ & \beta=90.50 \\ & V=1200.8 ; Z=2 \end{aligned}$ | $P 2{ }_{1} / c$ | 9: $\infty^{3}[(5: 4 \Delta+\mathrm{T})+(3: 2 \Delta+\mathrm{T})+(1: \mathrm{T})]$ | [12] |
| $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{8} \mathrm{O}_{13}$ | 4 | $\begin{aligned} & a=6.507 \\ & b=17.796 \\ & c=8.377 \\ & \beta=96.57 \\ & V=964 ; Z=4 \end{aligned}$ | $P 2_{1} / a$ | 8: $\infty^{3}[(5: 4 \Delta+\mathrm{T})+(3: 2 \Delta+\mathrm{T})]$ | [13-15] |
| $\beta-\mathrm{Na}_{2} \mathrm{~B}_{8} \mathrm{O}_{13}$ | 4 | $\begin{aligned} & a=11.731 \\ & b=7.880 \\ & c=10.410 \\ & \beta=99.883 \\ & V=948 ; Z=4 \end{aligned}$ | $P 2{ }_{1} / c$ | 8: $\infty^{3}[(5: 4 \Delta+\mathrm{T})+(3: 2 \Delta+\mathrm{T})]$ | [1] |

115); $\quad \varepsilon-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}: \quad a=18.862 \AA, \quad b=5.856 \AA, \quad c=$ $8.839 \AA, \quad \beta=93.82^{\circ}, \quad V=974.15 \AA^{3}, \quad Z=6, \quad F(30)=$ $4(0.046,171)$. It is worth noting that the results of automatic indexation of powder patterns are of poor quality. Moreover, all these unit cell parameters are completely different from those of $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ and $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ (Table 5). However, almost all of the reflections reported for the $\beta$ form can be indexed with $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$ unit cell parameters. It seems that out of the five forms reported for $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ [18], only two species, whose structures were solved, were clearly determined: triclinic $\alpha-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$, and the new orthorhombic borate $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$, which probably corresponds to the so-called $\beta-\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$. Other experiments of crystal growth are necessary to try to obtain single crystals of the other forms, $\gamma, \delta$ and $\varepsilon$ of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$, to verify the correct formula by the resolution of their structures.

### 3.4. Discussion

For the past 2 years, three new sodium borates have been found: $\beta-\mathrm{Na}_{2} \mathrm{~B}_{8} \mathrm{O}_{13}$ [1], $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}$ [2] and $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}$. Table 5 reports the main crystallographic and structural data of the anhydrous sodium borates whose structures were determined. Most of them participate in the complex equilibria existing in the $\mathrm{Na}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ binary phase diagram [3-6]. In a recent
paper [2], we pointed out that probably $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}$ $(\mathrm{B} / \mathrm{Na}=2.33)$ replaces $\mathrm{Na}_{2} \mathrm{~B}_{5} \mathrm{O}_{8.5}(\mathrm{~B} / \mathrm{Na}=2.5)$ in the phase diagram. However, with the discovery of the new borate $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5}(\mathrm{~B} / \mathrm{Na}=2.17)$, it was difficult to state that none of the formulas proposed in the phase diagram exist. In a recent work about $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ [6], to explain the complexity of the $\mathrm{Na}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ phase diagram around this composition, several thermal analyses were performed on samples preliminary annealed at different temperatures and for different periods. From the results, a small domain of solid solution under the melting point and another form of $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ (named $\beta$ ) were pointed out without any identification from X-ray powder diffraction [6]. However, the discovery of the new borate $\mathrm{Na}_{6} \mathrm{~B}_{13} \mathrm{O}_{22.5} \quad\left(68.5 \mathrm{~mol} \% \quad \mathrm{~B}_{2} \mathrm{O}_{3}\right)$ close to $\mathrm{Na}_{2} \mathrm{~B}_{4} \mathrm{O}_{7}$ ( $66.7 \mathrm{~mol} \% \mathrm{~B}_{2} \mathrm{O}_{3}$ ) and some experiments to grow single crystals suggests that this last compound melts incongruently while the new borate melts congruently around $740^{\circ} \mathrm{C}$. The $\mathrm{Na}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ phase diagram must be corrected to integrate this result as well as the presence of $\mathrm{Na}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}$ [2].

As in other $M_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ binary systems ( $M$ monovalent element), several possibilities may arise with $\mathrm{B} / M$ ratio, and not only with integer values; e.g., in the $\mathrm{Cs}_{2} \mathrm{O}-\mathrm{B}_{2} \mathrm{O}_{3}$ system several new cesium borates were found that led to the following B/Cs ratios: 1, 2, 2.33, 3, 4.33, 5,9 [38]. So, it appears that, from the association of $\mathrm{BO}_{3}$ triangles and $\mathrm{BO}_{4}$ tetrahedra, any complex
anions may exist as, e.g., in $\mathrm{K}_{5} \mathrm{~B}_{19} \mathrm{O}_{31}$ [39] or $\mathrm{Rb}_{5} \mathrm{~B}_{19} \mathrm{O}_{31}$ [40] $(\mathrm{B} / \mathrm{K}$ and $\mathrm{B} / \mathrm{Rb}=3.8)$, in $\mathrm{Cs}_{3} \mathrm{~B}_{13} \mathrm{O}_{21}(\mathrm{~B} / \mathrm{Cs}=4.33)$ (this compound has in fact 26 independent boron atoms) [41], in $\mathrm{Cs}_{3} \mathrm{~B}_{7} \mathrm{O}_{12}(\mathrm{~B} / \mathrm{Cs}=2.33)$ [42], and in the studied borate $\mathrm{Na}_{6} \mathrm{~B}_{12} \mathrm{O}_{22.5}(\mathrm{~B} / \mathrm{Na}=2.17)$ with following complex FBBs: 19: $\infty^{3} \quad 2[(5: \quad 4 \Delta+\mathrm{T})+(3: \quad 2 \Delta+\mathrm{T})+$ $(1: \Delta)+0.5(1: \mathrm{T})], \quad 26: \quad \infty^{2} \quad[2(5: \quad 4 \Delta+\mathrm{T})+4(3:$ $2 \Delta+\mathrm{T})+4(1: \Delta)], 63: \infty^{2}[(7: 3 \Delta+4 \mathrm{~T})+10(5: 3 \Delta+$ $2 \mathrm{~T})+((5: 2 \Delta+3 \mathrm{~T})+(1: \mathrm{T})$ or $(5: 3 \Delta+2 \mathrm{~T})+(1: \Delta))]$, and 13: $\left.\infty^{3}[(5: 3 \Delta+2 \mathrm{~T})+2(4: 2 \Delta+2 \mathrm{~T}))\right]$.

Experiments are in progress to study the nonlinear optical properties of this new noncentrosymmetric borate on powder and single crystal.

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[^1]:    Symmetry code:
    ${ }^{\mathrm{a}}-x+1.5,-y+0.5, z+0.5$.
    ${ }^{\mathrm{b}} x, y+1, z$.
    ${ }^{c} x, y, z+1$.
    ${ }^{\mathrm{d}}-x+1, y, z+0.5$.
    ${ }^{\mathrm{e}}-\mathrm{x}+1, y, z-0.5$.
    ${ }^{\mathrm{f}} x,-y+2, z-0.5$.
    ${ }^{\mathrm{g}} x, y, z-1$.
    ${ }^{\mathrm{h}} x,-y+1, z-0.5$.
    ${ }^{\mathrm{i}}-x+1.5, y+0.5, z$.
    ${ }^{\mathrm{j}} x, y+1, z-1$.
    ${ }^{\mathrm{k}}-x-1,-y+2, z+1$.
    ${ }^{1}-x+1.5,-y+1.5, z-0.5$.
    ${ }^{m} x,-y+2, z+0.5$.
    ${ }^{\mathrm{n}} x,-y+1, z+0.5$.

